

# Signals and Systems ECE 202

## Assignment 8

Document and submit your analytic results. Also generate and publish a MATLAB document, where appropriate, for the following exercises. Submit your MATLAB html folder and original MATLAB code (m files).

1. Calculate the mean-square-error between  $x(t) = \text{rect}(t/d)$  and its finite Fourier series  $x_M(t)$  truncated to the range  $-M \leq k \leq M$ . Let the duty cycle  $d = 0.5$ , and the period  $T = 1$ . The mean-square error (MSE) is defined as

$$\text{MSE} = \frac{1}{N} \sum_{t=-T/2}^{T/2} (x(t) - x_M(t))^2$$

where  $N$  is the number of time points evaluated.

Plot MSE vs  $M$  (use MATLAB function `semilogy`) and find  $M$  for which MSE is less than  $10^{-4}$ . For that value of  $M$ , find the maximum error and the location of the maximum error.

2. Find the Fourier series for the following function. Use MATLAB to plot your answer. Evaluate the degree of convergence as described in the first exercise.

$$x(t) = \begin{cases} 1 - (t/a)^2 & t \leq a \\ 0 & t > a \end{cases}$$

for  $-T/2 \leq t \leq T/2$  and  $a \leq T/2$ .

3. Find the Fourier series for the following function. Use MATLAB to plot your answer. Demonstrate for several values of  $a$ . Evaluate the degree of convergence as described in the first exercise.

$$x(t) = e^{-|at|}$$

where  $-T/2 \leq t \leq T/2$

4. Find the Fourier series for the following function. Use MATLAB to plot your answer. Evaluate the degree of convergence as described in the first exercise.

$$x(t) = t - t^3$$

where  $-1 \leq t \leq 1$ .

5. Given the following Fourier series with period  $T = 1$ . Use MATLAB to plot the series for  $-2.5 \leq t \leq 2.5$  for  $M = 9, 25,$  and  $75,$  and determine what time function is being represented.

$$x_M(t) = \sum_{k=-M}^M e^{j2\pi kt}$$

For reasons that should become clear, plot  $\frac{1}{2M+1}x_M(t)$ .

6. Using the CTFS table of transforms in Appendix E and the CTFS properties, find the CTFS harmonic function of each of these periodic signals using the representation period  $T_F$  indicated. Use MATLAB to plot your answer.

(a)  $x(t) = 5 [\text{tri}(t - 1) - \text{tri}(t + 1)] * \frac{1}{4}\text{comb}\left(\frac{t}{4}\right)$  with  $T_F = 4$

(b)  $x(t) = 3 \sin(6\pi t) + 4 \cos(8\pi t)$  and  $T_F = 1$

(c)  $x(t) = 2 \cos(24\pi t) - 8 \cos(30\pi t) + 6 \sin(36\pi t)$  with  $T_F = 2$