Signals and Systems ECE 202
Test 2 Solution

1. (a) $\text{rect}(x/d)$
   (b) $\exp(j6\pi t)$

2. The Fourier Series coefficients are given by

$$2\int_0^d \sin(2\pi kt)dt = -2\frac{\cos(2\pi kt)}{2\pi k}\bigg|_0^d = \frac{1-\cos(2\pi kd)}{\pi k}$$

3. (a) Let $h(t) = K\exp(-t/\tau)u(t)$

$$\tau\frac{dh(t)}{dt} + h(t) = \delta(t)$$

$$-K\exp(-t/\tau)u(t) + K\delta(t) + K\exp(-t/\tau)u(t) = \delta(t)$$

   so $h(t) = \exp(-t/\tau)u(t)$

   (b) Let

$$h(t) = K_1\exp(-t/\tau)u(t) + K_2\delta(t)$$

   Substitute into differential equation

$$\tau\frac{dh(t)}{dt} + h(t) = \tau\delta'(t)$$

$$-K_1\exp(-t\tau)u(t) + K_1\tau\delta(t) + K_2\tau\delta'(t) + K_1\exp(-t/\tau)u(t) + K_2\delta(t) = \tau\delta'(t)$$

   from which we conclude

$$h(t) = \delta(t) - \frac{1}{\tau}\exp(-t/\tau)u(t)$$