



# Standard Mathematical Tables

The publication of the CRC Standard Mathematical Tables takes full advantage of the reorganized, rewritten, and enhanced sections instituted in the 15th and 16th Editions. The popular reception accorded by users of these latter editions convinces The Chemical Rubber Co. that its continuing production of reference materials updated is a "must" and will be continued.

The sections on mensuration, trigonometry, analytic geometry, curves and graphs, the algebra of sets, determinants and matrices, expanded sections on statistics, and the inclusion of the hexadecimal and decimal conversion tables continue to be the strengths of this new 17th Edition.

## Seventeenth Edition

To meet the many requests of users of the 16th Edition, it has been decided to bring back a Five-place Logarithmic-Sine-Trigonometric table. One of the real contributions is the inclusion of a reorganization of the section on Integrals. One Hundred and Fifty useful integral formulae.

### Editor-in-chief of Mathematics

Staff is indebted to Dr. SAMUEL M. SELBY, Ph.D. Sc.D., Board, who undertook this project.

SAMUEL M. SELBY, Ph.D. Sc.D.

Distinguished Professor Emeritus of Mathematics and formerly Chairman, Mathematics Department, University of Akron.

As in Presently Chairman, Mathematics Department, Hiram College, Hiram, Ohio numbers of the Advisory Board, who are listed in the forefront of this edition, for their continued cooperation. Constructive suggestions from interested users of this edition as well as preceding ones are important and are welcome at all times since they serve as a feed-back which continually assists and guides the publication efforts of The Chemical Rubber Co. It helps to maintain the high degree of accuracy that the CRC Standard Mathematical Tables has enjoyed for its many editions.

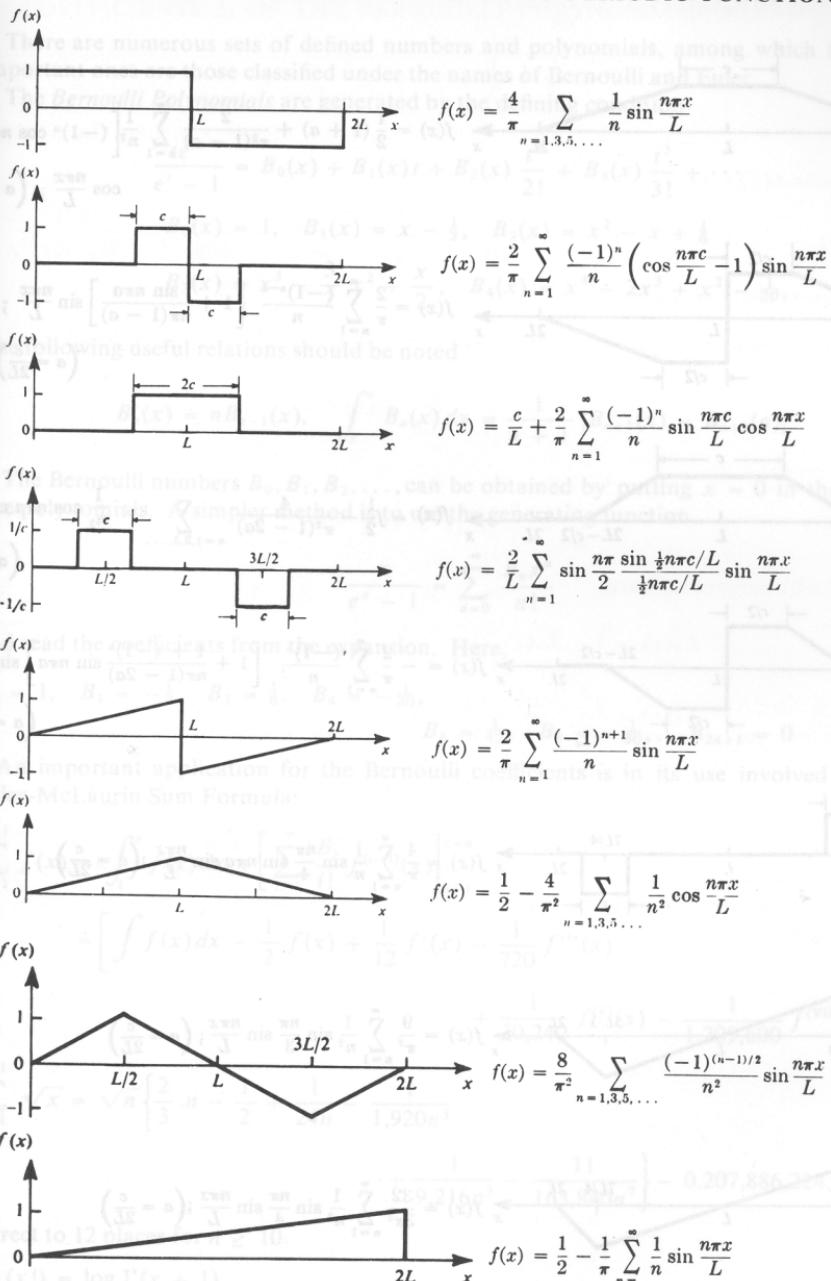
SAMUEL M. SELBY, Editor-in-Chief, Mathematics

ROBERT C. WEAST, Editor-in-Chief

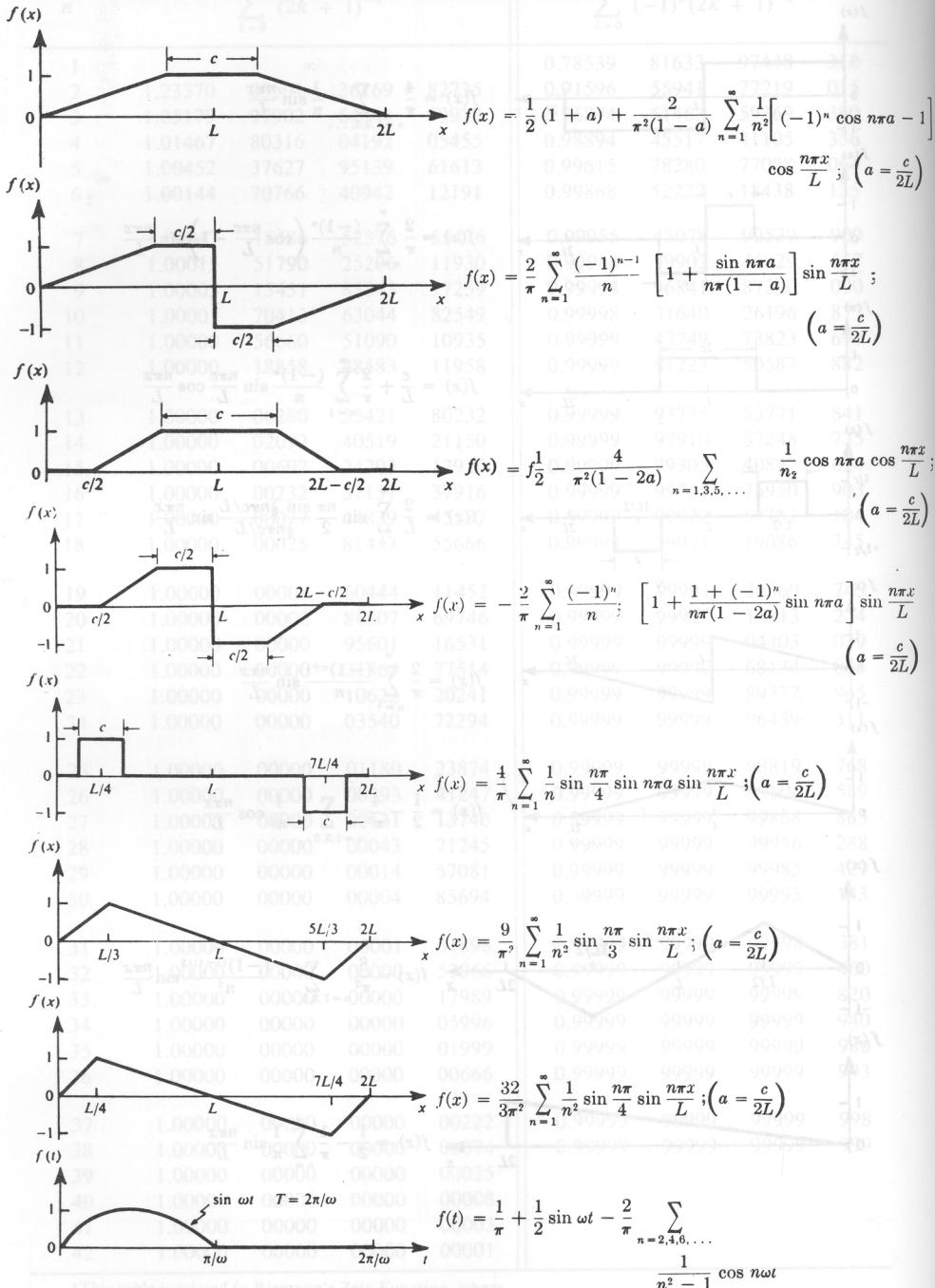
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## (b) FOURIER EXPANSIONS FOR BASIC PERIODIC FUNCTIONS



## FOURIER EXPANSIONS FOR BASIC PERIODIC FUNCTIONS (Continued)



\*This table is related to Riemann's Zeta Function, where

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^x - 1} dx = \zeta(s)$$

## FOURIER TRANSFORMS FINITE SINE TRANSFORMS

	$f_s(n)$	$F(x)$
1	$f_s(n) = \int_0^\pi F(x) \sin nx dx \quad (n = 1, 2, \dots)$	$F(x)$
2	$(-1)^{n+1} f_s(n)$	$F(\pi - x)$
3	$\frac{1}{n}$	$\frac{\pi - x}{\pi}$
4	$\frac{(-1)^{n+1}}{n}$	$\frac{x}{\pi}$
5	$\frac{1 - (-1)^n}{n}$	1
6	$\frac{2}{n^2} \sin \frac{n\pi}{2}$	$\begin{cases} x & \text{when } 0 < x < \pi/2 \\ \pi - x & \text{when } \pi/2 < x < \pi \end{cases}$
7	$\frac{(-1)^{n+1}}{n^3}$	$\frac{x(\pi^2 - x^2)}{6\pi}$
8	$\frac{1 - (-1)^n}{n^3}$	$\frac{x(\pi - x)}{2}$
9	$\frac{\pi^2(-1)^{n-1}}{n} - \frac{2[1 - (-1)^n]}{n^3}$	$x^2$
10	$\pi(-1)^n \left( \frac{6}{n^3} - \frac{\pi^2}{n} \right)$	$x^3$
11	$\frac{n}{n^2 + c^2} [1 - (-1)^n e^{c\pi}]$	$e^{cx}$
12	$\frac{n}{n^2 + c^2}$	$\frac{\sinh c(\pi - x)}{\sinh c\pi}$
13	$\frac{n}{n^2 - k^2} \quad (k \neq 0, 1, 2, \dots)$	$\frac{\sin k(\pi - x)}{\sin k\pi}$
14	$\begin{cases} \frac{\pi}{2} & \text{when } n = m \\ 0 & \text{when } n \neq m \end{cases} \quad (m = 1, 2, \dots)$	$\sin mx$
15	$\frac{n}{n^2 - k^2} [1 - (-1)^n \cos k\pi] \quad (k \neq 1, 2, \dots)$	$\cos kx$
16	$\begin{cases} \frac{n}{n^2 - m^2} [1 - (-1)^{n+m}] & \text{when } n \neq m = 1, 2, \dots \\ 0 & \text{when } n = m \end{cases}$	$\cos mx$
17	$\frac{n}{(n^2 - k^2)^2} \quad (k \neq 0, 1, 2, \dots)$	$\frac{\pi \sin kx}{2k \sin^2 k\pi} - \frac{x \cos k(\pi - x)}{2k \sin k\pi}$
18	$\frac{b^n}{n} \quad ( b  \leq 1)$	$\frac{2}{\pi} \arctan \frac{b \sin x}{1 - b \cos x}$
19	$\frac{1 - (-1)^n}{n} b^n \quad ( b  \leq 1)$	$\frac{2}{\pi} \arctan \frac{2b \sin x}{1 - b^2}$

## FINITE COSINE TRANSFORMS

	$f_c(n)$	$F(x)$
1	$f_c(n) = \int_0^\pi F(x) \cos nx dx \quad (n = 0, 1, 2, \dots)$	$F(x)$
2	$(-1)^n f_c(n)$	$F(\pi - x)$
3	0 when $n = 1, 2, \dots$ ; $f_c(0) = \pi$	1
4	$\frac{2}{n} \sin \frac{n\pi}{2}$ ; $f_c(0) = 0$	$\begin{cases} 1 & \text{when } 0 < x < \pi/2 \\ -1 & \text{when } \pi/2 < x < \pi \end{cases}$
5	$-\frac{1 - (-1)^n}{n^2}; f_c(0) = \frac{\pi^2}{2}$	$x$
6	$\frac{(-1)^n}{n^2}; f_c(0) = \frac{\pi^2}{6}$	$\frac{x^2}{2\pi}$
7	$\frac{1}{n^2}; f_c(0) = 0$	$\frac{(\pi - x)^2}{2\pi} - \frac{\pi}{6}$
8	$3\pi^2 \frac{(-1)^n}{n^2} - 6 \frac{1 - (-1)^n}{n^4}; f_c(0) = \frac{\pi^4}{4}$	$x^3$
9	$\frac{(-1)^n e^c \pi - 1}{n^2 + c^2}$	$\frac{1}{c} e^{cx}$
10	$\frac{1}{n^2 + c^2}$	$\cosh c(\pi - x)$
11	$\frac{k}{n^2 - k^2} [(-1)^n \cos \pi k - 1] \quad (k \neq 0, 1, 2, \dots)$	$\sin kx$
12	$\frac{(-1)^{n+m} - 1}{n^2 - m^2}; f_c(m) = 0 \quad (m = 1, 2, \dots)$	$\frac{1}{m} \sin mx$
13	$\frac{1}{n^2 - k^2} \quad (k \neq 0, 1, 2, \dots)$	$-\frac{\cos k(\pi - x)}{k \sin k\pi}$
14	0 when $n = 1, 2, \dots$ ; $f_c(m) = \frac{\pi}{2} \quad (m = 1, 2, \dots)$	$\cos mx$

The following functions are called FOURIER SINE TRANSFORMS<sup>1</sup>.

### FOURIER SINE TRANSFORMS<sup>1</sup>

$F(x)$	(a) Definition	$f_s(\alpha)$
1 $\begin{cases} 1 & (0 < x < a) \\ 0 & (x > a) \end{cases}$		$\sqrt{\frac{2}{\pi}} \left[ \frac{1 - \cos \alpha}{\alpha} \right]$
2 $x^{p-1} (0 < p < 1)$		$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p)}{\alpha^p} \sin \frac{p\pi}{2}$
3 $\begin{cases} \sin x & (0 < x < a) \\ 0 & (x > a) \end{cases}$		$\frac{1}{\sqrt{2\pi}} \left[ \frac{\sin[a(1-\alpha)]}{1-\alpha} - \frac{\sin[a(1+\alpha)]}{1+\alpha} \right]$
4 $e^{-x}$		$\sqrt{\frac{2}{\pi}} \left[ \frac{\alpha}{1+\alpha^2} \right]$
5 $xe^{-x^2/2}$		$\alpha e^{-\alpha^2/2}$
6 $\cos \frac{x^2}{2}$		$\sqrt{2} \left[ \sin \frac{\alpha^2}{2} C\left(\frac{\alpha^2}{2}\right) - \cos \frac{\alpha^2}{2} S\left(\frac{\alpha^2}{2}\right) \right]^*$
7 $\sin \frac{x^2}{2}$		$\sqrt{2} \left[ \cos \frac{\alpha^2}{2} C\left(\frac{\alpha^2}{2}\right) + \sin \frac{\alpha^2}{2} S\left(\frac{\alpha^2}{2}\right) \right]^*$

\*  $C(y)$  and  $S(y)$  are the Fresnel integrals

$$C(y) = \frac{1}{\sqrt{2\pi}} \int_0^y \frac{1}{\sqrt{t}} \cos t dt,$$

$$S(y) = \frac{1}{\sqrt{2\pi}} \int_0^y \frac{1}{\sqrt{t}} \sin t dt.$$

<sup>1</sup> More extensive tables of the Fourier sine and cosine transforms can be found in Fritz Oberhettinger, "Tabelle zur Fourier Transformation," Springer (1957).

### FOURIER COSINE TRANSFORMS

$F(x)$	(a) Definition	$f_c(\alpha)$
1 $\begin{cases} 1 & (0 < x < a) \\ 0 & (x > a) \end{cases}$		$\sqrt{\frac{2}{\pi}} \frac{\sin \alpha x}{\alpha}$
2 $x^{p-1} (0 < p < 1)$		$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p)}{\alpha^p} \cos \frac{p\pi}{2}$
3 $\begin{cases} \cos x & (0 < x < a) \\ 0 & (x > a) \end{cases}$		$\frac{1}{\sqrt{2\pi}} \left[ \frac{\sin[a(1-\alpha)]}{1-\alpha} + \frac{\sin[a(1+\alpha)]}{1+\alpha} \right]$
4 $e^{-x}$		$\sqrt{\frac{2}{\pi}} \left( \frac{1}{1+\alpha^2} \right)$
5 $e^{-x^2/2}$		$e^{-\alpha^2/2}$
6 $\cos \frac{x^2}{2}$		$\cos \left( \frac{\alpha^2}{2} - \frac{\pi}{4} \right)$
7 $\sin \frac{x^2}{2}$		$\cos \left( \frac{\alpha^2}{2} + \frac{\pi}{4} \right)$

FOURIER TRANSFORMS<sup>1</sup>

	$F(x)$	$f(\alpha)$
1	$\frac{\sin ax}{x}$	$\begin{cases} \sqrt{\frac{\pi}{2}} &  \alpha  < a \\ 0 &  \alpha  > a \end{cases}$
2	$\begin{cases} e^{ipx} & (p < x < q) \\ 0 & (x < p, x > q) \end{cases}$	$\frac{i}{\sqrt{2\pi}} \frac{e^{ip(w+\alpha)} - e^{iq(w+\alpha)}}{(w + \alpha)}$
3	$\begin{cases} e^{-cx + iwx} & (x > 0) \\ 0 & (x < 0) \end{cases}$	$\frac{i}{\sqrt{2\pi}(w + \alpha + ic)}$
4	$e^{-px^2} \quad R(p) > 0$	$\frac{1}{\sqrt{2p}} e^{-\alpha^2/4p}$
5	$\cos px^2$	$\frac{1}{\sqrt{2p}} \cos \left[ \frac{\alpha^2}{4p} - \frac{\pi}{4} \right]$
6	$\sin px^2$	$\frac{1}{\sqrt{2p}} \cos \left[ \frac{\alpha^2}{4p} + \frac{\pi}{4} \right]$
7	$ x ^{-p} \quad (0 < p < 1)$	$\sqrt{\frac{2}{\pi}} \frac{\Gamma(1-p) \sin \frac{p\pi}{2}}{ \alpha ^{(1-p)}}$
8	$\frac{e^{-a x }}{\sqrt{ x }}$	$\frac{\sqrt{\sqrt{(a^2 + \alpha^2)} + a}}{\sqrt{a^2 + \alpha^2}}$
9	$\frac{\cosh ax}{\cosh \pi x} \quad (-\pi < a < \pi)$	$\sqrt{\frac{2}{\pi}} \frac{\cos \frac{a}{2} \cosh \frac{\alpha}{2}}{\cosh \alpha + \cos a}$
10	$\frac{\sinh ax}{\sinh \pi x} \quad (-\pi < a < \pi)$	$\frac{1}{\sqrt{2\pi}} \frac{\sin a}{\cosh \alpha + \cos a}$
11	$\begin{cases} \frac{1}{\sqrt{a^2 - x^2}} & ( x  < a) \\ 0 & ( x  > a) \end{cases}$	$\sqrt{\frac{\pi}{2}} J_0(a\alpha)$
12	$\frac{\sin [b\sqrt{a^2 + x^2}]}{\sqrt{a^2 + x^2}}$	$\begin{cases} 0 & ( \alpha  > b) \\ \sqrt{\frac{\pi}{2}} J_0(a\sqrt{b^2 - \alpha^2}) & ( \alpha  < b) \end{cases}$
13	$\begin{cases} P_n(x) & ( x  < 1) \\ 0 & ( x  > 1) \end{cases}$	$\frac{i^n}{\sqrt{\alpha}} J_{n+\frac{1}{2}}(\alpha)$
14	$\begin{cases} \frac{\cos [b\sqrt{a^2 - x^2}]}{\sqrt{a^2 - x^2}} & ( x  < a) \\ 0 & ( x  > a) \end{cases}$	$\sqrt{\frac{\pi}{2}} J_0(a\sqrt{a^2 + b^2})$
15	$\begin{cases} \frac{\cosh [b\sqrt{a^2 - x^2}]}{\sqrt{a^2 - x^2}} & ( x  < a) \\ 0 & ( x  > a) \end{cases}$	$\sqrt{\frac{\pi}{2}} J_0(a\sqrt{\alpha^2 - b^2})$

<sup>1</sup>More extensive tables of Fourier transforms can be found in W. Magnus and F. Oberhettinger, "Formulas and Theorems of the Special Functions of Mathematical Physics," pp. 116-120. Chelsea (1949).