Digital Signal Processing I ECE 561
Fall 1997 Test 1

1. Given definitions of linearity, shift-invariance, stability, causality, and memoryless as applied to systems. (10 points)

2. Explain how to obtain the magnitude and phase of a complex vector in Matlab. (5 points)

3. Given the continuous signal $\text{sinc}^2(x/10)$. Find its bandwidth and the minimum sampling frequency that satisfies the Nyquist requirement. (10 points)

4. Find the convolution of [1 2 -3 4 -6] with [0 0 1 2 4 1]. (10 points)

5. Demonstrate the response of a linear system to a stochastic input.

   (a) Generate a random signal as follows

   ```
   xr = rand(8192,1);
   xr = xr - mean(xr);
   ```

   Calculate the variance of this signal and compare to the theoretical value of 1/12. (5 points)

   (b) Find the autocorrelation of the signal using

   ```
   [xc lags] = xcorr(xr,50);
   stem(lags,xc);
   ```

   Interpret the resulting plot. What is its expected mathematical form? (5 points)

   (c) Find the output signal for the following linear system and the cross correlation between input and output.

   ```
   a = [1 -0.9];
   b = 1;
   y = filter(b,a,xr);
   [yc lags] = xcorr(xr,y,100);
   ```

   What is the mathematical expression of the impulse response of the system? (5 points)

   (d) Find the mathematical expression for the cross correlation ϕ_{xy} as calculated by `xcorr`. Superimpose a plot of this mathematical expression on top of the data points from `yc`. Explain what normalization was required. (10 points)

6. The Fourier transform of the sequence $x[n] = a^n u[n]$ is given by

$$X(\omega) = \frac{1}{1 - ae^{-j\omega}}$$

The real, imaginary, and magnitude of the frequency response is plotted on p. 55 for ($a = 0.9$ and $a = 0.5$). Is the scale of these figures correct? What values of a were probably used? See if you can do a better job of making and labeling these figures. (10 points)

7. Show that the Fourier Transform of a moving average filter (Eq. 2.111) with $M_1 = M_2$ is equivalent to the Matlab function `diric`($2\pi x$,n) where $n = M_1 + M_2 + 1$. Compare this function to `sinc`(nx) for $n = 5$ and $n = 11$ over the range ($0 \leq x \leq 1$). Compare to $S(x) = \text{sinc}(nx - 1) + \text{sinc}(nx) + \text{sinc}(nx + 1)$.(10 points)

8. Generate the frequency response and impulse response of the following filter. (10 points)

$$y[n] + 0.13y[n-1] + 0.52y[n-2] + 0.3y[n-3] = 0.16x[n] - 0.48x[n-1]$$

9. Find the Fourier transform of

$$w[n] = \begin{cases} \frac{1}{2}\left(1 + \cos(2\pi n/M)\right) & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

for $M = 3$ (10 points)