Digital Signal Processing I ECE 561
Fall 1997 Test 3

1. Given a digital filter with a pole at $r = 0.75$ and $\theta = 30$ degrees from the horizontal axis in the z-plane.

   (a) Find the location of a second pole such that the resulting impulse response is real. (5 points)

   (b) Find the location of a zero such that the resulting filter is all-pass (but with complex coefficients for the difference equation). (5 points)

2. Given the digital filter described by

$$H(z) = \frac{(1-3z^{-1})(1+2z^{-1}+4z^{-2})(2-z^{-1})}{(1-0.5z^{-1})(1+0.4z^{-1})}$$

   (a) Generate the pole-zero plot for the filter and tell if this transfer function can be a stable, causal system. (5 points)

   (b) Plot the group delay as a function of (angular) frequency. (5 points)

   (c) Transform the filter to a minimum phase filter and show the corresponding pole-zero plot. (5 points)

   (d) Plot the group delay function of the resulting filter (5 points)

3. Design a filter to remove a 60 Hz spurious component and its harmonics from a signal sampled at 8000 Hz. Your filter must attenuate these frequencies by a factor of 100, and must attenuate no more than 0.95 for frequencies 0.2% above or below the target frequencies (i.e. $\pm$ 1.2 Hz at 60 Hz). Show a plot of the magnitude of the response vs. frequency centered about 60 Hz. (10 points)

4. Given the filter

$$H(z) = 1 + 0.5z^{-1} - 0.3z^{-2} + 0.3z^{-3} - 0.5z^{-5} - z^{-6}$$

   Is this a linear FIR filter? If so, identify its type, and find its amplitude function $A(\omega)$ in factored form. For example, if the filter were type II, write the amplitude function as

$$A(\omega) = \cos(\omega/2)\sum_{k=0}^{K} g[k]\cos(k\omega)$$

   (10 points)

5. Design a 9th-order low-pass Chebyshev type II with a cutoff frequency of 2000 Hz and 1 dB of ripple in the stop band. Assume the signal is sampled at 8000 Hz.
   (a) Plot its pole-zero plot. (5 points)
   (b) Plot its magnitude response. (5 points)
   (c) Find the maximum group delay within the passband. (5 points)
   (d) Generate its State-Space representation. (5 points)
6. Design a FIR filter using a Kaiser window with the same basic specs as above.
   (a) Plot the impulse response. (5 points)
   (b) Plot the magnitude response. (5 points)
   (c) Find the maximum group delay within the passband. (5 points)
7. Design an FIR filter whose frequency response approximates a triangle function of the form

$$A(f) = \begin{cases} 1 - f/f_0 & f \leq f_0 \\ 0 & f > f_0 \end{cases}$$

   where f_0 is 3000 Hz and the sampling frequency is 8000 Hz. Plot the magnitude response of your filter. (15 points)