Digital Signal Processing I ECE 561
Fall 1997 Test 2

1. Find the z-transform of the following signals (5 points each):

   (a) $f[k] = \frac{1}{k!}u[k]$

   (b) $g[n] = 4n(1-n)u[k]$

   Note that

   $$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

2. Find the inverse z-transform of the following functions (5 points each);

   (a) $X(z) = (1+2z^{-2})(1-4z^{-1})(1+z)$

   (b) $X(z) = \log(\frac{1}{1-az^{-1}}), \qquad |z| > |a|.$

3. The sequence

   $$q[n] = 4\frac{n}{T}\left(1 - \frac{n}{T}\right) \qquad 0 \le n \le T$$

   is a sampled parabola (assume T= 30). Apply the filter $H(z) = 1 - z^{-1}$ as an approximation of a derivative. Verify that the second derivative is constant and equal to -8. What happens at the start of the sequence? (10 points)

4. Given the following stable, causal system

   $$H(z) = \frac{2 + 2.7z^{-1} - 0.36z^{-2}}{1 + 0.5z^{-1} - 0.36z^{-2}}$$

   (5 points per item)

   (a) Find the poles and zeros and identify the region of convergence

   (b) Evaluate h[0].

   (c) Find the response of the system to the unit-step signal $u[n]$.

   (d) Verify that the DC gain (the limit of the step response as $n \to \infty$) is obtained by evaluating $H(z)$ at $z = 1$.

5. Design a one-pole low pass digital filter with a bandwidth of 4 Hz for signals sampled at 8 KHz. The DC gain should be unity. Verify the performance using Matlab by plotting the frequency response and the group delay. (10 points)

6. Given the following sequence [1 -2 3 4 -5 1 2 -1], where the first point is $n = 0$. (5 points per item)

   (a) Assuming that the sequence is periodic (N=8), find the magnitude and phase of the first harmonic.

   (b) Find the cyclic autocorrelation of the sequence.

   (c) Find the cyclic selfconvolution of the sequence.

   (d) Find the linear autocorrelation of the sequence.

   (e) Find the linear selfconvolution of the sequence.

   (f) Show how to use the fft routine in Matlab to construct the linear selfconvolution sequence.

   In each case, identify the $n = 0$ point of the output sequence.

7. Download the file 3.au from the class web page. Identify the relative magnitudes and frequencies (in Hz) of the dominant spectral components present in the signal. Give an estimate of the precision of your answer. (10 points)