Digital Signal Processing I ECE 561
Fall 2003 Test 1

1. Given definitions of linearity, shift-invariance, stability, causality, and memoryless as applied to systems. (10 points)

2. Explain how to obtain the magnitude and phase of a complex vector in Matlab. (5 points)

3. Given the continuous signal $\text{sinc}^2(t/T)$ where T is 5 μs. Find its bandwidth and the minimum sampling frequency that satisfies the Nyquist requirement. (10 points)

4. Find the convolution of [1 6 -3 -4 6 -1] with [1 0 3 1 4]. (10 points)

5. Download the file `sample2003.zip` from the class web page. The sine wave sounds about right, but there is something very wrong with the square and triangle waves. Describe the problem and how to fix it. (10 points)

6. The Fourier transform of the sequence $x[n] = a^n u[n]$ is given by

$$X(\omega) = \frac{1}{1 - ae^{-j\omega}}$$

   The real part, imaginary part, magnitude and phase of the frequency response is plotted on Figure 2.22 (p. 57-58) for ($a = 0.9$ and $a = 0.5$). Are these figures correct (look at ω =0)? What values of a were probably used? Create a similar set of (correct) figures for $a = 0.5$ and a = -0.5. (10 points)

7. Determine if the systems described by the following equations are linear or nonlinear (5 points)

   (a) $y[n] = nx[n]$

   (b) $y[n] = x[n^2]$

   (c) $y[n] = (x[n])^2$

   (d) $y[n] = Ax[n] + B$, where A and B are not zero.

   (e) $y[n] = e^{x[n]}$

8. Determine if the systems described by the following equations are causal or noncausal (5 points)

   (a) $y[n] = x[n] - x[n-1]$

   (b) $y[n] = Ax[n] + B$, where A and B are not zero.

(c) $y[n] = x[n] + 3x[n+4]$

(d) $y[n] = x[2n]$

(e) $y[n] = x[-n]$

9. Given a system described by the following difference equation.

$$y[n] + \frac{1}{15}y[n-1] - \frac{2}{5}y[n-2] = x[n]$$

(a) Write the analytic equation for the transfer function $H(e^{j\omega})$. (5 points)

(b) Expand the transfer function as a partial fraction and then inverse transform it to find the analytic expression for the impulse response. Calculate the first three values. (5 points)

(c) Use `y = filter(b,a,x)` where $x[n] = \delta[n]$ in Matlab to plot the impulse response. List the first three values. (5 points)

(d) Find the general form of the homogeneous solution. (5 points)

(e) Use Matlab to plot a particular solution for $x[n] = (3/5)^n u[n]$. List the first five values (5 points)

(f) Find the discrete time Fourier transform of the $x[n]$ defined above. (5 points)

(g) Find an analytic form of the particular solution for the $x[n]$ defined above. Calculate the first five values (5 points)