Digital Signal Processing I ECE 561
Fall 2003 Test 2

Download the file `test2data.zip` from the class web page.

1. The sound file `tone2.wav` contains three principal sinuosoidal components. Use the FFT to find the frequencies and the relative strengths of these components. Find the frequency resolution of your calculation (sample spacing in frequency domain). (10 points)

2. Given the function $f(x) = x \cdot \text{rect}(x)$ (a linear ramp for $-1/2 \leq x \leq 1/2$), use the FFT for the following.

   (a) Plot the real and imaginary parts of the spectrum. (5 points)

   (b) Plot the self convolution $(f(x) * f(x))$ of $f(x)$. (5 points)

   (c) Plot the autocorrelation of $f(x)$. (5 points).

3. Use the function $f(x)$ (assume x is seconds) to construct a periodic signal $p(x)$ of period 3 sec, and total length 30 cycles, sampled at 100 Hz.

   (a) Plot the magnitude of the spectrum. (5 points)

   (b) Plot the autocorrelation function of $p(x)$. (5 points)

4. Given $p(x) = \text{rect}(x/c) * \text{rect}(x/d)$, find the Fourier series corresponding to $p(x) * \text{comb}(x)$. Verify your answer by means of an example. (10 points)

5. Given the sound file `signal2.wav`, consisting of a periodic signal buried in noise, find the period of the underlying signal. (10 points) For extra credit (5 points) extract and plot one cycle of the periodic component of the signal.

6. Find the z-transform of the following signals (5 points each):

   (a) $f[k] = \frac{1}{k!}u[k]$

   (b) $g[n] = 4n(1-n)u[k]$

   Note that

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

7. Find the inverse z-transform of the following functions (5 points each);

   (a) $X(z) = (1+2z^{-2})(1-4z^{-1})(1+z)$

   (b) $X(z) = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}, \qquad |z| > \frac{1}{2}.$

8. Given a causal LTI system with the system function

$$H(z) = \frac{1-4z^{-1}+2z^{-2}-4z^{-3}+z^{-4}}{(1-\frac{1}{6}z^{-1}-\frac{1}{3}z^{-2})}$$

   Use Matlab to

   (a) Find the zero-pole representation. (5 points)

   (b) Plot the pole-zero diagram. (5 points)

   (c) Find impulse response of the system. (5 points)

   (d) Find the partial fraction expansion. (5 points)

   (e) Plot the frequency response and phase verus frequency. ((5 points)