Digital Signal Processing I ECE 561
Fall 2004 Test 2

1. Find the z-transform of the following signals (5 points each):

   (a) $f[k] = \frac{1}{k!}u[k]$

   (b) $g[n] = n(n-1)u[n]$

   Note that

   $$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

2. Find the inverse z-transform of the following functions (5 points each);

   (a) $X(z) = (1 + 4z^{-2})(1 - 2z^{-1} + z^{-2})(1 + z)$

   (b) $X(z) = \frac{4-3z^{-1}+3z^{-2}}{(z+2)(z-3)^2}, \quad 2 < |z| < 3.$

3. Given a causal LTI system with the system function

   $$H(z) = 0.0534 \frac{(1 + z^{-1})(1 - 1.0166z^{-1} + z^{-2})}{(1 - 1.499z^{-1} + 0.8482z^{-2})(1 - 1.5548z^{-1} + 0.6493z^{-2})}$$

   Use Matlab to

   (a) Plot the magnitude response and phase verus frequency. What type of filter does this transfer function prepresent? (5 points)

   (b) Convert this system to state-space representation. (5 points)

   (c) Plot the pole-zero diagram. (5 points)

   (d) Find the partial fraction expansion. (5 points)

   (e) Find the second-order sections. (5 points)

4. Given a digital filter with a pole at $r = 0.6$ and $\theta = 20$ degrees from the horizontal axis in the z-plane.

   (a) Find the location of a second pole such that the resulting impulse response is real. Find the transfer function (b and a) (5 points)

   (b) Find the location of the zeros such that the resulting filter is all-pass. Find the transfer function (b and a) (5 points)

5. Given the digital filter described by

$$H(z) = \frac{(1 - 3z^{-1})(1 + 2z^{-1} + 4z^{-2})(2 - z^{-1})}{(1 - 0.5z^{-1})(1 + 0.4z^{-1})}$$

   (a) Generate the pole-zero plot for the filter and tell if this transfer function can be a stable, causal system. (5 points)

   (b) Transform the filter to a minimum phase filter and show the corresponding pole-zero plot. (5 points)

   (c) Plot the group delay as a function of (angular) frequency for both filters (on the same plot). (5 points)

6. Given the filter

$$H(z) = -c_4 - c_3 z^{-2} - c_2 z^{-4} - c_1 z^{-6} + c_1 z^{-8} + c_2 z^{-10} + c_3 z^{-12} + c_4 z^{-14}$$

   where $c_1 = 0.6121882$, $c_2 = 0.137338$, $c_3 = 0.0324802$, and $c_4 = 0.0073304$. Is this a linear FIR filter? If so, identify its type, and find its amplitude function $A(\omega)$ in factored form. For example, if the filter were type II, write the amplitude function as

$$A(\omega) = \cos(\omega/2) \sum_{k=0}^{K} g[k] \cos(k\omega)$$

   Plot the magnitude of the filter response over the range ω = -π to π. (10 points)

7. Design an elliptic analog bandpass filter with the following specifications: passband edges at 20 kHz and 45 kHz, stopband edges at 10 kHz and 60 kHz, peak passband ripple of 0.5 dB, and a minimum stopband attenuation of 40 dB. Show the order of your design, plot the gain response, and verify that the filter meets its specifications. (20 points).