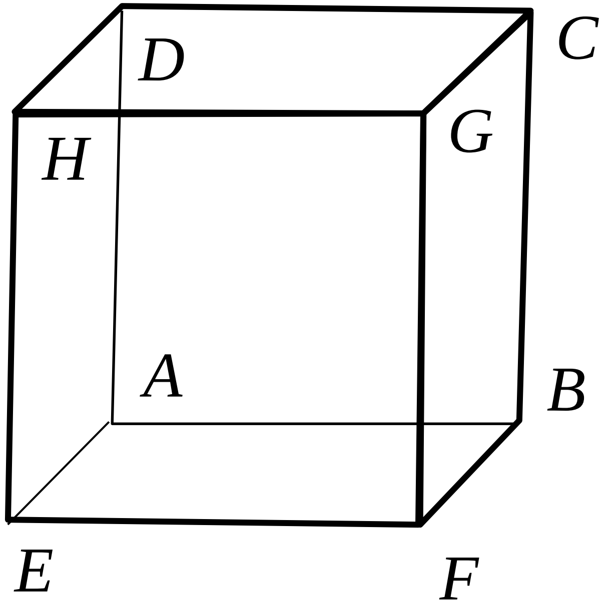
Boundary Representation



Vertices	Edges
A (0, 0, 0)	AB
B (1, 0, 0)	BC
C (1, 1, 0)	CD
D (0, 1, 0)	DA
E (0, 0, 1)	EF
F (1, 0, 1)	FG
G (1, 1, 1)	GH
H (0, 1, 1)	HE
	AE
	BF
	CG
	DH

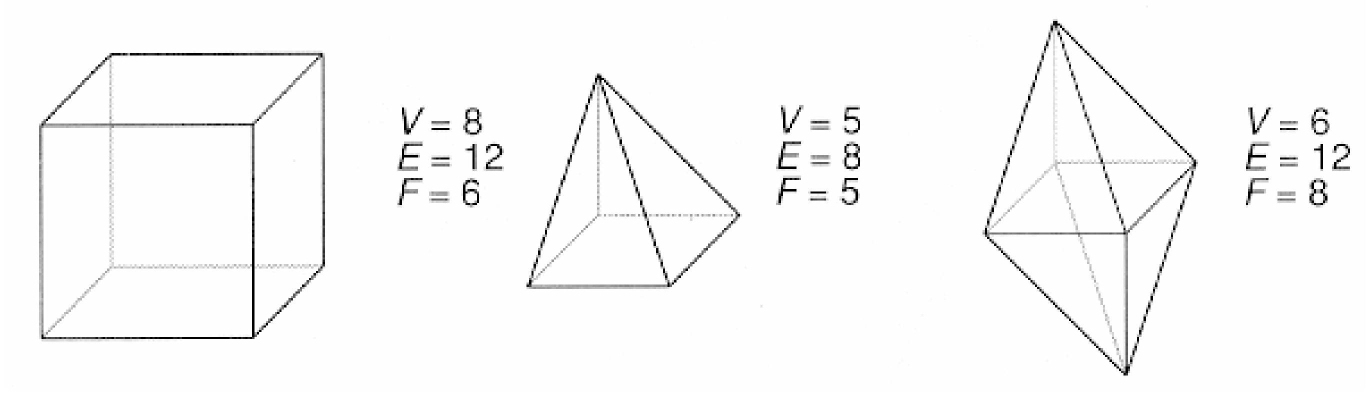
Reference:

James D. Foley, Andries van Dam, Steven Feiner, and John Hughes, *Computer Graphics*, *Principles and Practice*, *Second Edition*, Addison-Wesley Publishing Company (1990) ISBN 0-201-12110-7

Euler’s Formula

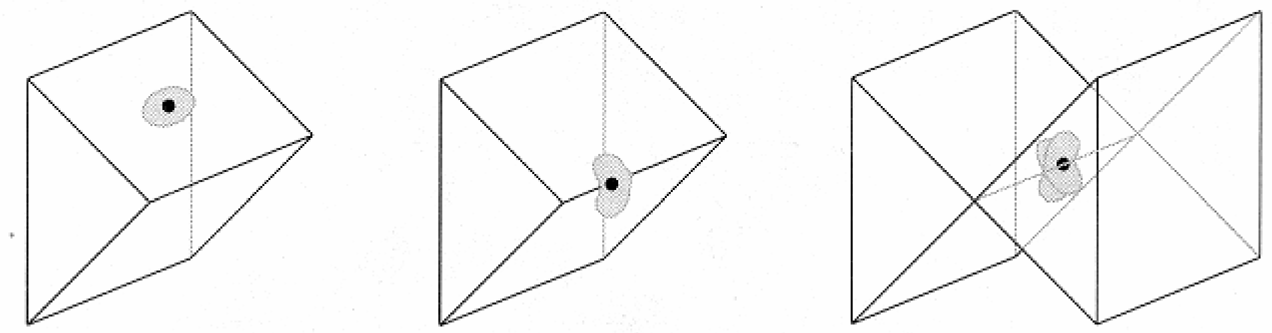
A *polyhedron* is a solid that is bounded by a set of faces (polygons) whose edges are shared by an even number of faces (exactly two in the case of 2-manifolds).
A *simple polyhedron* is one that can be deformed into a sphere. Simple polyhedrons have no holes. Compare a *sphere* to a *torus*.
The boundary representation of a simple polyhedron satisfies Euler's formula, which expresses an invariant relationship among the number of faces *F*, edges *E*, and vertices *V*:

$$V - E + F = 2$$



2-Manifolds

By definition, every point on a *2-manifold* has some arbitrarily small neighborhood of points around it that can be considered topologically the same as a disk in the plane.
Each point, shown below as a black dot, has a neighborhood of surrounding points that is a topological disk, as shown in gray in (a) and (b). Object (c) is not a 2-manifold because it has points for which this is not true.



(a) (b) (c)

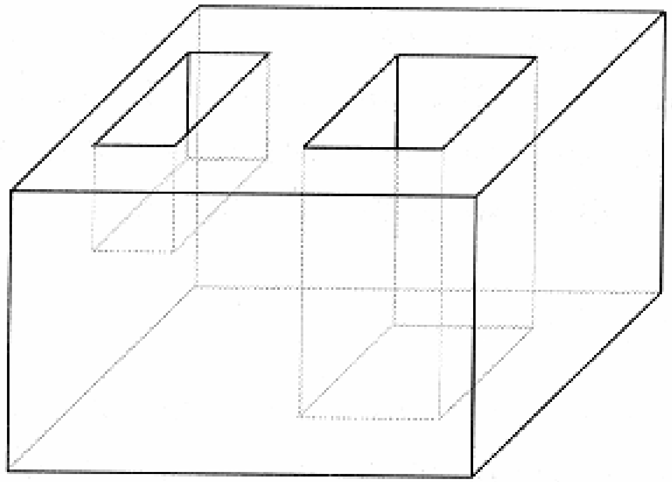
Generalized Euler’s Formula

A generalization of Euler's formula applies to
2-manifolds that have faces with holes

$$V - E + F - H = 2\left(C - G\right)$$

V	number of vertices
F	number of faces
E	number of edges
H	number of holes
G	number of holes that pass through the solid
C	number of components (parts) of the object

Example



$$V - E + F - H = 2(C - G)$$

V	E	F	H	C	G
24	36	15	3	1	1

The object above is considered to have two holes in its top face and one hole in its bottom face.

If an object has a single component, its G is known as its *genus*.