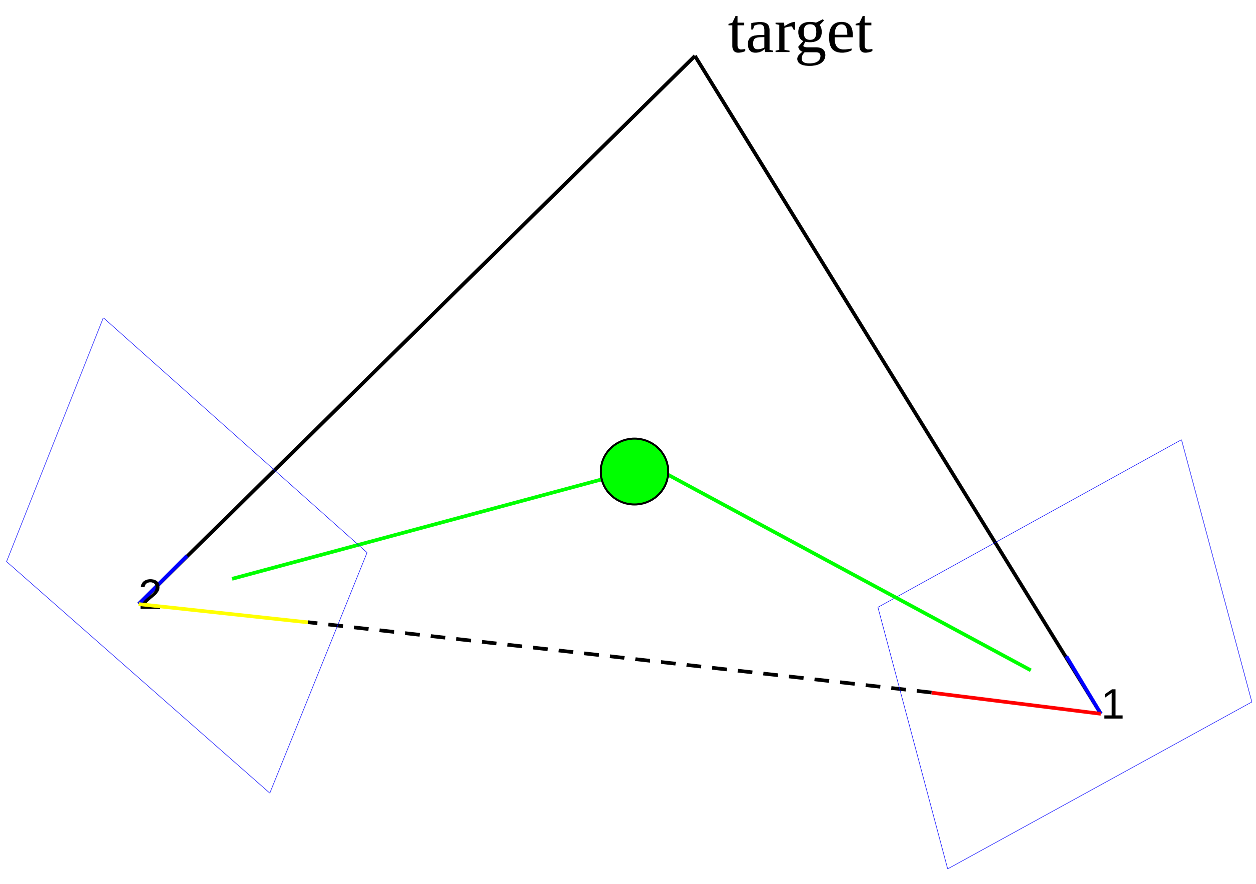
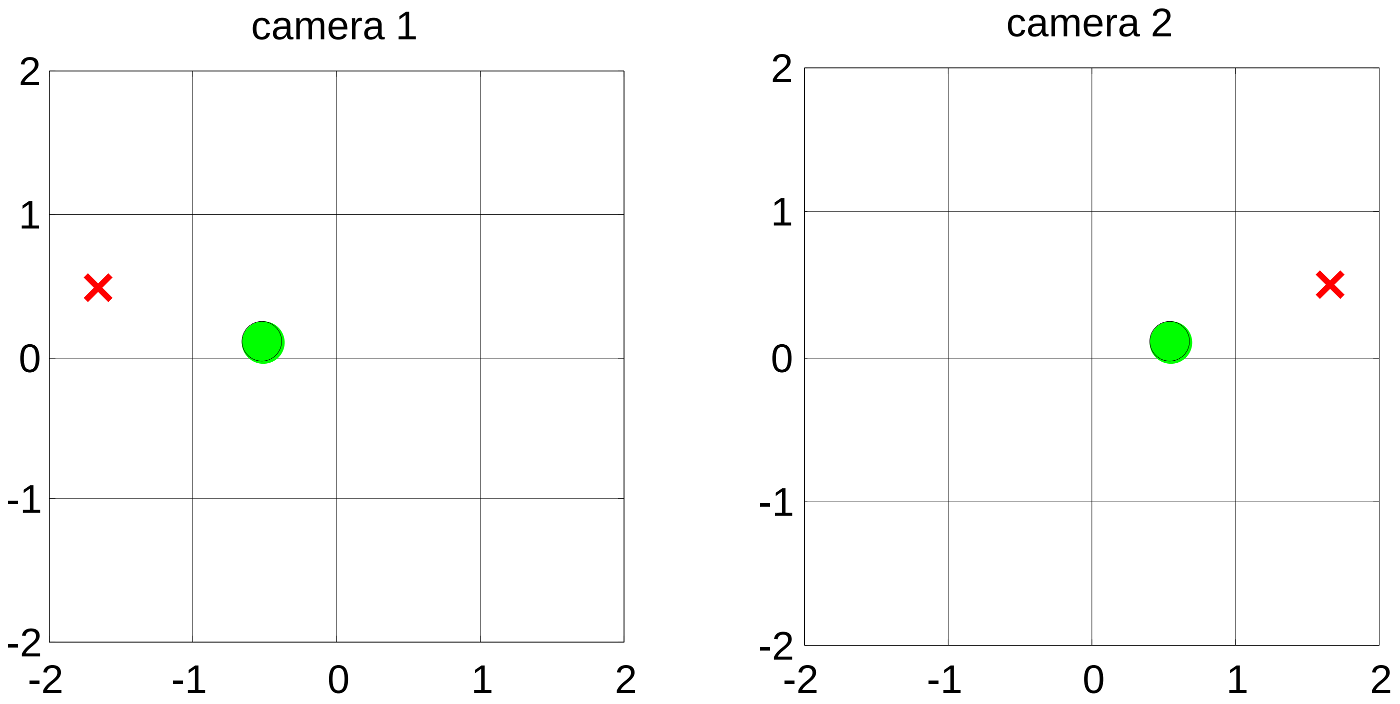
Stereo View



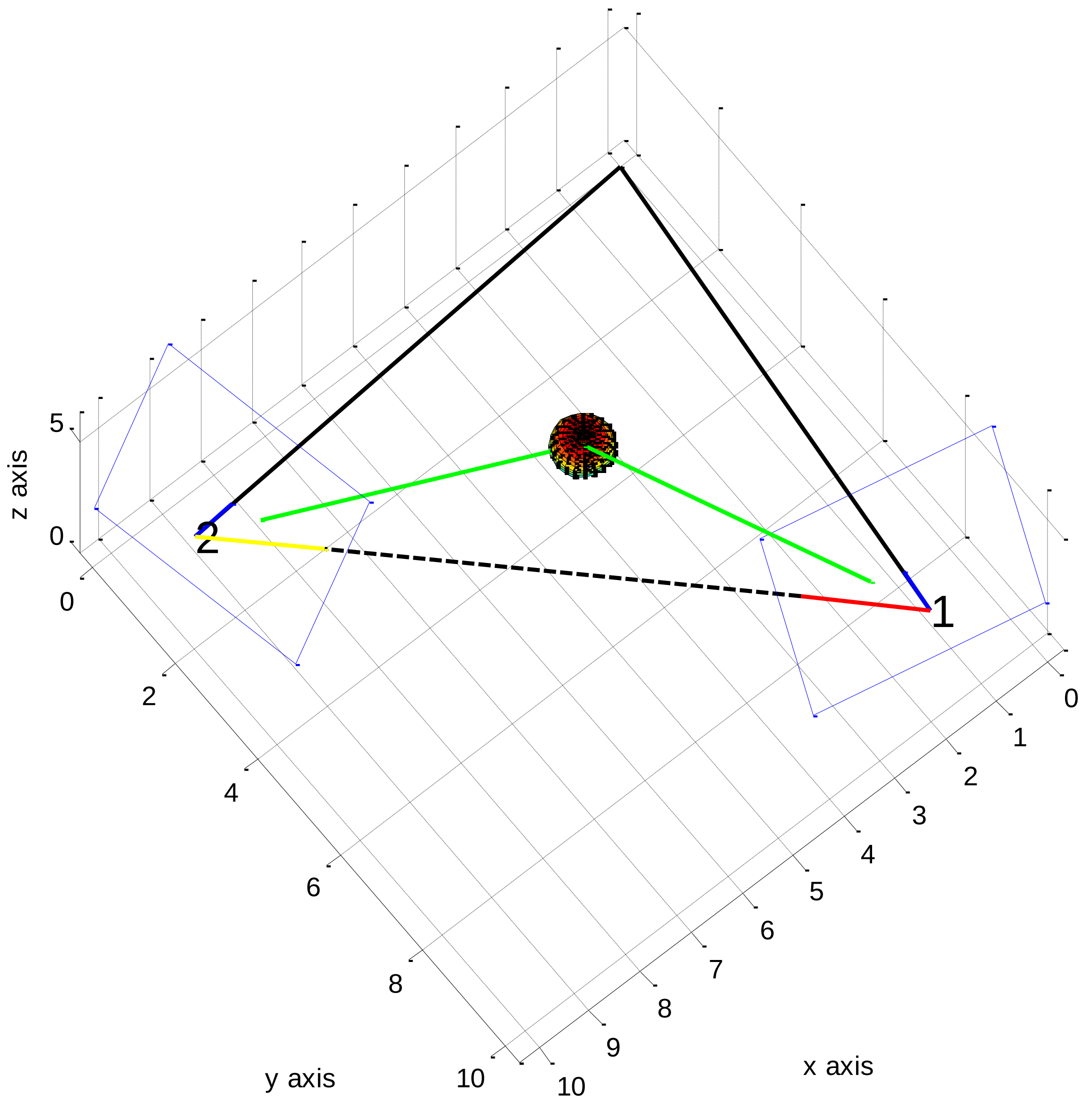
Camera View



projected point

epipole

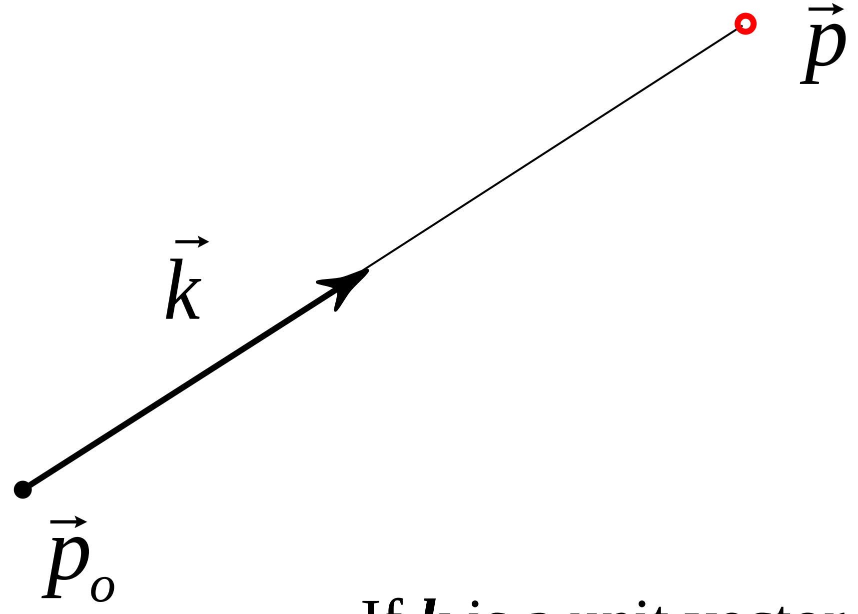
Stereo View (with axes)



3D Line

A 3D line can be defined *parametrically*.

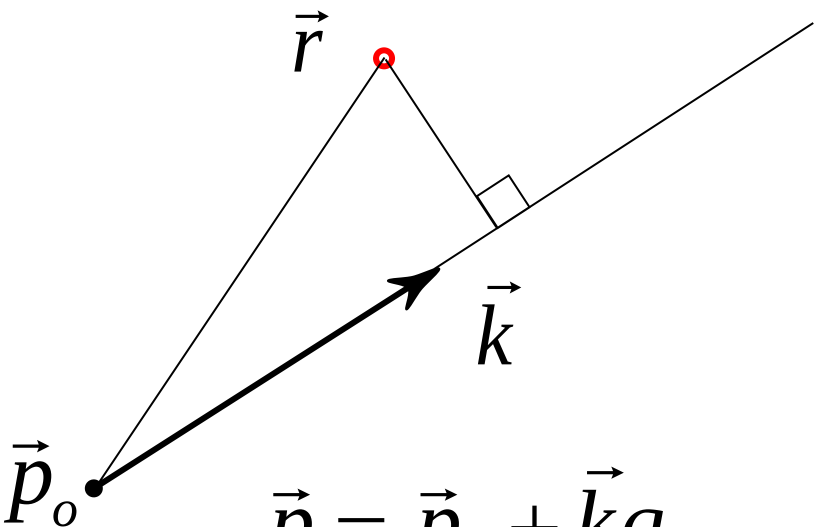
$$\vec{p} = \vec{p}_o + \vec{k}q$$



If $\boldsymbol{k}$ is a unit vector, then q is the distance from $\boldsymbol{p}_o$ to $\boldsymbol{p}$.

Distance from Point to Line

The shortest distance from a point to a line is the perpendicular vector ***d***.



$$\vec{p} = \vec{p}_o + \vec{k}q$$

$$\vec{d} = \vec{r} - \vec{p}$$

$$\vec{k} \cdot \vec{d} = 0$$

$$\vec{k} \cdot (\vec{r} - \vec{p}) = \vec{k} \cdot \left(\vec{r} - \vec{p}_o - \vec{k}q\right) = 0$$

The parametric distance q to the point of closest approach on the line is

$$q = \frac{\vec{k} \cdot (\vec{r} - \vec{p}_o)}{\vec{k} \cdot \vec{k}}$$

Back Projection

$$\vec{P}_1 = \vec{C}_1 + \vec{k}_1 p \qquad q = \frac{\vec{k} \cdot (\vec{r} - \vec{p}_o)}{\vec{k} \cdot \vec{k}}$$

$$\vec{P}_2 = \vec{C}_2 + \vec{k}_2 q$$

$$p\vec{k}_1 \cdot \vec{k}_1 = \vec{k}_1 \cdot \left(\vec{P}_2 - \vec{C}_1\right)$$

$$q\vec{k}_2 \cdot \vec{k}_2 = \vec{k}_2 \cdot \left(\vec{P}_1 - \vec{C}_2\right)$$

$$p\vec{k}_1 \cdot \vec{k}_1 = \vec{k}_1 \cdot \left(\vec{C}_2 + \vec{k}_2 q - \vec{C}_1\right)$$

$$q\vec{k}_2 \cdot \vec{k}_2 = \vec{k}_2 \cdot \left(\vec{C}_1 + \vec{k}_1 p_1 - \vec{C}_2\right)$$

$$p\vec{k}_1 \cdot \vec{k}_1 - q\vec{k}_1 \cdot \vec{k}_2 = \vec{k}_1 \cdot \left(\vec{C}_2 - \vec{C}_1\right)$$

$$-p\vec{k}_1 \cdot \vec{k}_2 + q\vec{k}_2 \cdot \vec{k}_2 = -\vec{k}_2 \cdot \left(\vec{C}_2 - \vec{C}_1\right)$$

Suppose projections are parallel

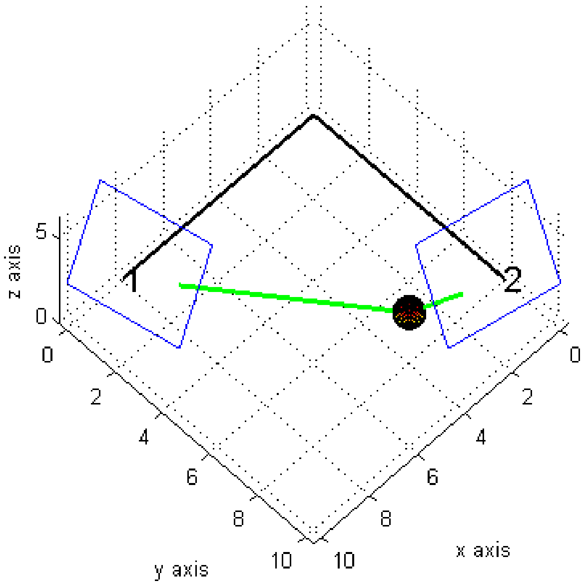
$$p\vec{k}_1 \cdot \vec{k}_1 - q\vec{k}_1 \cdot \vec{k}_2 = \vec{k}_1 \cdot \left(\vec{C}_2 - \vec{C}_1\right)$$
$$q\vec{k}_2 \cdot \vec{k}_2 - p\vec{k}_1 \cdot \vec{k}_2 = \vec{k}_2 \cdot \left(\vec{C}_1 - \vec{C}_2\right)$$

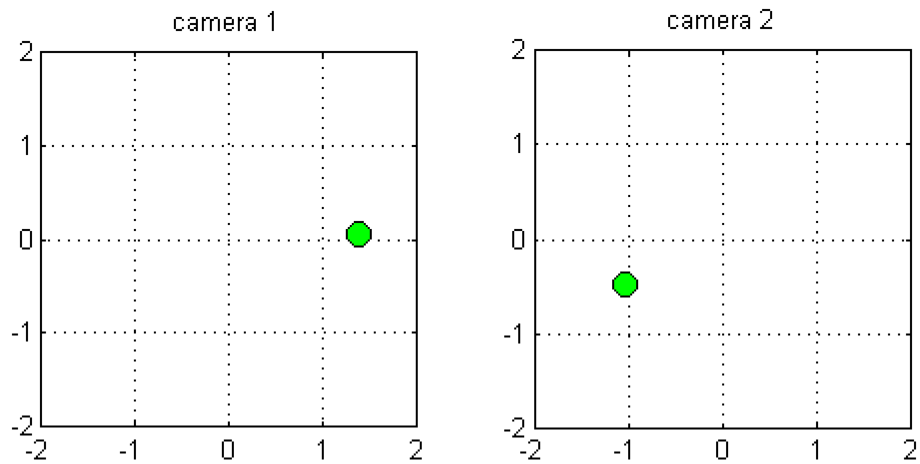
$$\left.\begin{aligned} p\vec{k} \cdot \vec{k} - q\vec{k} \cdot \vec{k} = \vec{k} \cdot \left(\vec{C}_2 - \vec{C}_1\right) \\ q\vec{k} \cdot \vec{k} - p\vec{k} \cdot \vec{k} = \vec{k} \cdot \left(\vec{C}_1 - \vec{C}_2\right) \end{aligned}\right\} \text{Same equation}$$

$$p - q = \frac{\vec{k} \cdot \left(\vec{C}_2 - \vec{C}_1\right)}{\vec{k} \cdot \vec{k}}$$

No unique solution

MATLAB Verification






```
k1 = invpt1 - cam1.pos;
k2 = invpt2 - cam2.pos;
C = cam2.pos - cam1.pos;
A = [dot(k1,k1) -dot(k1,k2); dot(k1,k2) -dot(k2,k2)];
b = [dot(k1,C); -dot(k2,C)];
q = A\b;

P1 = cam1.pos + q(1)*k1;
P2 = cam2.pos + q(2)*k2;
P = 0.5*(P1+P2);
```

projected point P = [4 8 3]

|P2-P1| = 0

error = 0

Projective Geometry

$$\begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\begin{bmatrix} a_2 & b_2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_2 & b_2 \end{bmatrix} \begin{bmatrix} -b_1 \\ a_1 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

Projective Geometry

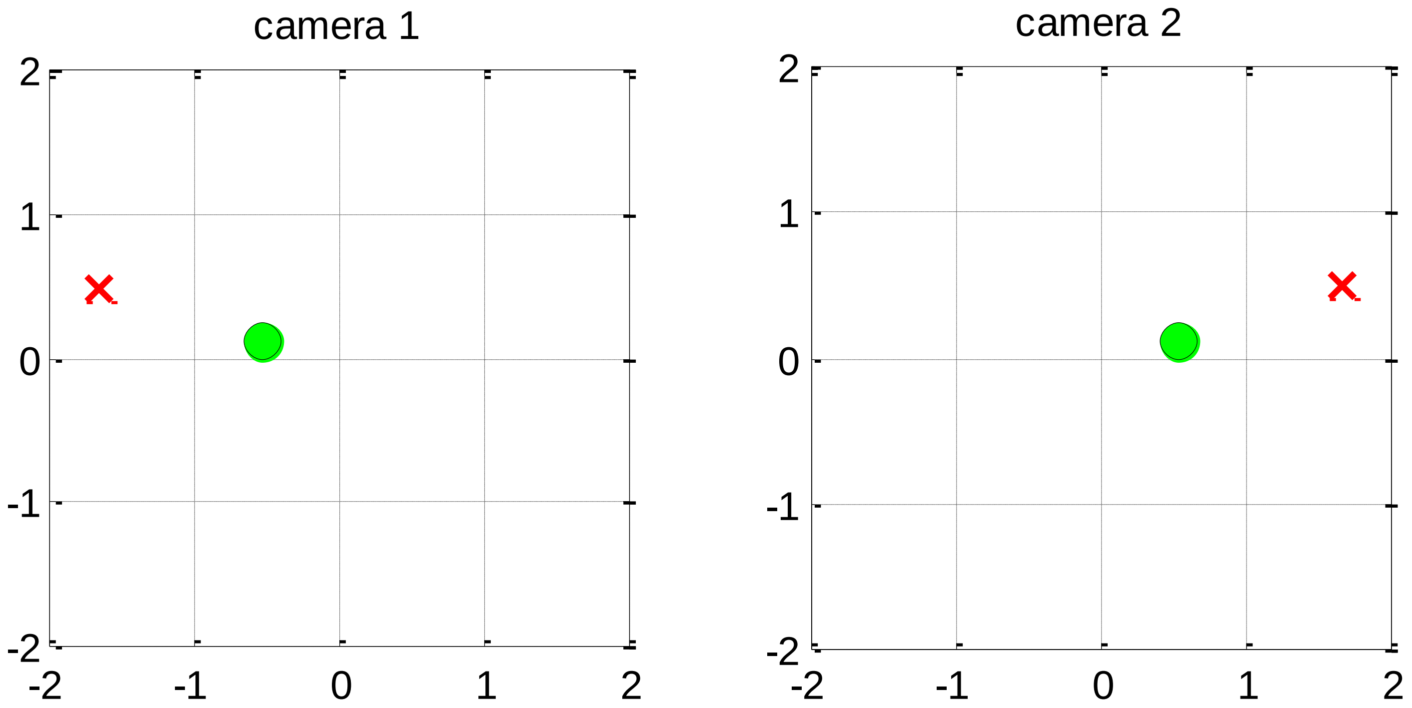
$$\begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\begin{bmatrix} a_2 & b_2 \end{bmatrix} \begin{bmatrix} -b_1 \\ a_1 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

Camera View



Stereo View